PROBABILISTIC MODEL AND ANALYSIS OF CONVENTIONAL PREINSTALLED MINE FIELD DEFENSE

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OF CONVENTIONAL PREINSTALLED
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by

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Simple models for a defense consisting of a preinstalled mine field possibly defended by an anti-tank weapon are derived and analyzed. This paper user a special Poisson process to model the one or two positions of mines in the mine field. The duel between the anti-tank weapon and offensive tanks crossing the field is modeled with a continuous time Markov chain. Some algebraic solutions and numerical results are obtained for specific scenarios.

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Probabilistic Model and Analysis of Conventional Preinstalled Mine Field Defense

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ABSTRACT

Simple models for a defense consisting of a preinstalled mine field possibly defended by an anti-tank weapon are derived and analyzed. This paper uses a special Poisson process to model the one or two positions of mines in the mine field. The duel between the anti-tank weapon and offensive tanks crossing the field is modeled with a continuous time Markov chain. Some algebraic solutions and numerical results are obtained for specific scenarios.

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I. INTRODUCTION

For more than two decades, conventional mine fields have been in place along the Demilitarized Zone and in front of strategic points in Korea. The way in which the effectiveness of the mine field has been increased and maintained is the periodic replacement of old mines and the addition of new mines to the field.

In the initial phase of a war, the enemy will attempt to use a quick attack to break through the defensive lines, like the one which was constructed by Korea along the D.M.Z., in order to seize some preplanned areas. For this operation, the enemy will use their numerical superiority in tanks and artillery. If the enemy is able to seize areas which are important politically and economically, the advance will stop.

There are many ways to improve and reinforce the defensive line. Against the enemy's quick tank and armored vehicle attack, the best way is to have a mine field that is defended by anti-tank weapons.

History and experience have shown that a mine field defended by an anti-tank weapon is more effective than a mine field by itself or the anti-tank weapon by itself. The reason why the mine field is the best means of defense against the enemy's tanks and armored vehicles is that

it may restrict the movement of enemy tanks and vehicles and thereby increase the effectiveness of anti-tank weapons. A mine field also assists in protecting friendly forces from sudden attack.

In this thesis, it is assumed that the preinstalled mines in a mine field act their characteristics with 100 percent reliability. Some simple probabilistic models are studied for scenarios of a mine field and a mine field defended by an anti-tank weapon. Algebraic and numerical results for some cases are provided.

II. SCENARIO ASSUMPTION

A. TERRAIN AND MINE FIELD

The mine field consists of both anti-personnel mines and anti-tank mines. The mine field is located crossing the likely axis of enemy advance.

The defensive forces are able to view possible offensive movement over the entire field. The entire field is also within the effective firing range of the defensive forces. It will be assumed that the offensive tanks have no maneuverability problems in the field. The positions of preinstalled mines are well camouflaged and are of the pressure-activated type. The offensive tanks cannot visually detect the mines and hence have no ability to avoid the mines.

B. OFFENSIVE FORCES

A limited objective for the offensive forces is to seize the defending positions. The reconnaissance of the offensive tanks did not provide enough information about their combat area to determine the defensive positions, but the defensive barriers are assumed to be placed along the axis of advance.

The mission of an anti-tank unit is to create a gap in the mine field and to destroy the defensive crew-served

weapons or tanks defending the field; these defending weapons are considered to be a major obstacle for following offensive forces. It is assumed that the offensive tanks will meet the mine field in a deployed formation.

C. DEFENSIVE FORCES

The defending forces are occupying preselected strong points where they can cover the likely axis of advances and also protect the mine field. Each anti-tank weapon's mission is to kill the enemy's armored vehicles or tanks in his assigned area. The anti-tank weapon is in a camouflaged fixed bunker which has usually one crenel. There are many bunkers for the crew-served weapons and anti-tank weapons in one strong point. Hence, even if the offensive tanks find a crenel, there may not be an anti-tank weapon there.

The dead ground (path) in the area will be covered by the friendly artillery firing. Thus, the offensive tanks must pass through the mine field. Usually, the defender wants to fire at the offensive tanks which are in the mine field, because the mines tend to restrict the maneuverability of an offensive tank and thereby make it easier for the defender to kill the tank.

For simplicity, an offensive tank which successfully crosses the mine field does not att~~r~~ack further defensive fire; that is, the duel between defense and offense will stop at the time when one offensive tank gets through the mine field.

III. MODEL ASSUMPTION

A. MINE FIELD

The preinstalled mine field has W units width and D units depth with a rectangular shape.

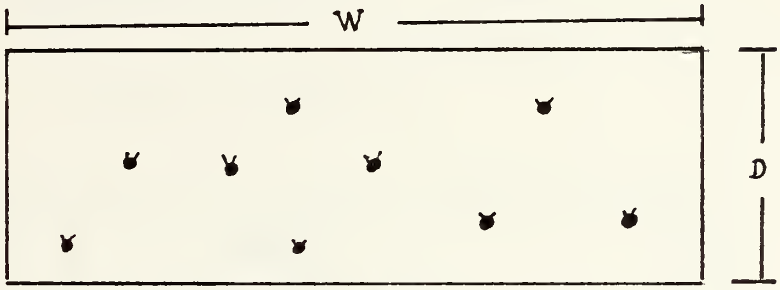

Figure 1. Mine Field Model

We assume the positions of pressure-activated anti-tank mines in the field form a spatially homogeneous Poisson process with the rate of "r" [Ref. 3]; that is, the number of mines in disjoint paths are independent random variables and the distribution of the anti-tank mines in the area of a path is Poisson with mean "$r|A|$," where $|A|$ is the area path $|A|$ [Ref. 4]. Assume that the track width of a tank is W_t units.

Let T_1 be the position of the first mine that a tank encounters in its path and J(A) be the number of mines in its path. The probability that there is no mine in the path equals

$$P\{T_1 > D\} = P\{J(A) = 0\} = e^{-r(W_t D)} \equiv e^{-RD}, \text{ where } R = r \cdot W_t.$$

Similarly, the probability of tank gets small Z units into the mine field without encountering a mine is

$$P\{T_1 > z\} = P\{J(z) = 0\} = e^{-Rz}$$

for $0 \leq z \leq D$.

Then

$$P\{T_1 \leq z\} = 1 - e^{-Rz}$$

for $0 \leq z \leq D$.

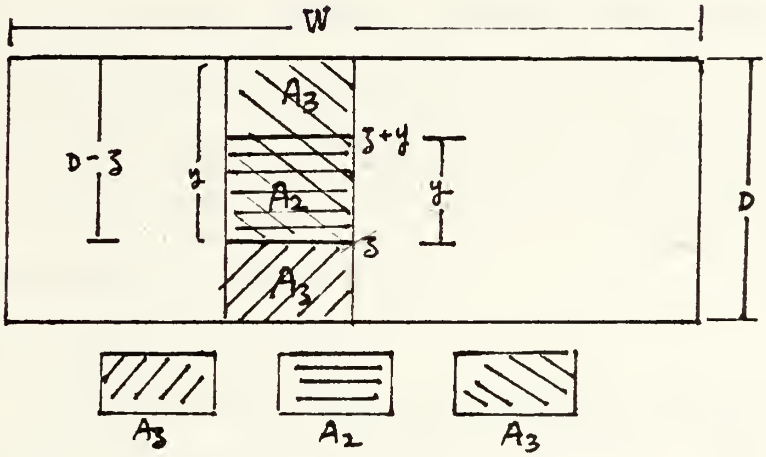

Figure 2. Area of a Path

Let T_2 be the position of the second mine in the path;

For $z < D$

$$P\{T_2 - T_1 > y \mid T_1 = z\} = P\{J(A_2) = 0\} = e^{-Ry}$$

for $z + y < D$

and

$$P\{T_2 - T_1 > y \mid T_1 = z\} = P\{J(A_3) = 0\} = e^{-R(D-z)}$$

for $z + y > D$

So T_1, T_2, T_3, ... have the same distribution as the arrival times of a Poisson process with rate "R" that stop at time D.

If any tank which gets into the mine field to attack the defensive encounters a mine, the results may be one of the following categories:

(0) No Damage

This category of damage neither excludes the tank from the combat nor limits its mobility or any other operations.

(1) Slight Damage

The slight damage category also does not affect the operational characteristics of the tank, but it increases the vulnerability of the tank to serious damage; for example, it is more likely that the tank will be completely destroyed when it encounters the next mine.

(2) Loss of Mobility

This category includes damage to the tank's track or demolition of its suspension system. The tank is expected to participate in the duel with the defender until it is killed by the anti-tank weapon.

(3) Completely Destroyed Damage

This category is defined as total destruction of the tank functions; that is, loss of mobility, loss of firepower, etc. A tank with this category of damage has no more influence on the field.

Let X_n be the class of damage to the tank due to its n^{th} encounter with a mine. The X_n can then take on the following values:

0; No damage

1; Slight damage

2; Loss of mobility

3; Completely destroyed damage (3.1)

assume $X_o = 0$.

Assume that:

X_n+1 is conditionally independent of X_o, X_1, X_2 ...X_{n-1}, given X_n. The transition probabilities are given as follows:

$$P\{X_1 = i \mid X_0 = 0\} = \begin{cases} 0; & i = 0 \\ P_{0i}; & i = 1, 2, 3. \end{cases}$$

$$P\{X_2 = j \mid X_1 = i\} = \begin{cases} 1; & i = 3,\ j = 3 \\ 1; & i = 1,\ j = 3 \\ 0; & \text{otherwise} \end{cases} \tag{3.2}$$

Let X(z) be the state of a tank due to mine encounters z units of distance into the mine field. The process {X(z); $0 \le z \le D$} is a continuous time Markov chain.

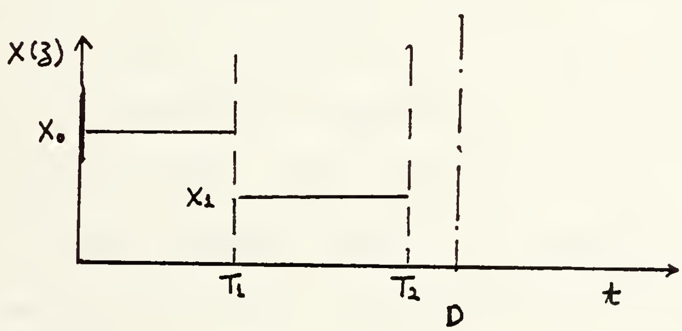

Figure 3. Distribution of Ti's

with absorbing states 3 (completely destroyed) and 2 (loss of mobility). The probability that a tank gets through the mine field is

$$P\{X(D) \in (0, 1)\}$$

by assumption (3.2).

$$P\{X(D) = 0\} = e^{-RD}$$

$$P\{X(D) = 1\} = (R \cdot D \cdot e^{-RD}) P_{01} .$$

Hence, the probability of a single tank getting through the mine field is:

$$P\{X(D) \in (0, 1)\} = e^{-RD} + (R \cdot D \cdot e^{-RD}) P_{01}$$

$$= (1 + R \cdot D \cdot P_{01}) e^{-RD} .$$

B. DUEL

When tanks appear in front of the mine field, they are seen by the defensive forces and the defender's anti-tank weapon starts to aim at one of the tanks. In general, the probability that the anti-tank weapon hits the offensive tank depends on the type of anti-tank weapon, the firing range, and the ground condition. In this thesis, the hit probability is assumed to depend only on the defender's anti-tank weapon type. The interval of the firing time (the time between rounds fired) is a random variable. The anti-tank weapon of the defender has the advantage of first firing.

The offensive tank leader is mainly involved in observing the battle area rather than trying to detect mines. His task is also to detect the defender's crew-served weapon positions. As assumed in the previous section, since the position of the anti-tank weapon is well camouflaged, the offensive tanks cannot begin to detect the anti-tank weapon position until the anti-tank weapon fires and gives away the region it is in. The probability of detecting the anti-tank weapon position depends on the distance; that is, the further away the defender's weapon is, the less likely its position will be detected when it fires. If any one of the offensive tanks detects the anti-tank weapon position or is hit by a mine, the warning and information are given to the other offensive tanks. When the offensive tanks find a potential anti-tank weapon position, it is uncertain whether the offensive tanks have detected the anti-tank weapon or not. Here it is assumed the offensive tanks can detect the position of the anti-tank weapon immediately after the anti-tank weapon's first firing. If an offensive tank hits an anti-tank weapon, the anti-tank weapon is completely destroyed.

After the defender's anti-tank weapon first fires, we assume that there is a duel between the defensive anti-tank weapon and the offensive tanks with constant hit probabilities P_A and P_T respectively, that is,

let

$$H_n(T) = \begin{cases} 1; & \text{if an offensive tank hits the anti-tank weapon during } n^{th} \text{ round.} \\ 0; & \text{otherwise.} \end{cases}$$

Also,

$$H_n(A) = \begin{cases} 1; & \text{if the anti-tank weapon hits an offensive tank during } n^{th} \text{ round.} \\ 0; & \text{otherwise.} \end{cases}$$

Assume $\{H_n(T); n = 1, 2, \ldots\}$ is a discrete time Markov chain with absorbing state 1,

$$P\{H_n(T) = 1 \mid H_{n-1}(T) = 1\} = 1$$

$$P\{H_n(T) = 1 \mid H_{n-1}(T) = 0\} = P_T \tag{3.3}$$

Assume $\{H_n(A); n = 1, 2, \ldots\}$ is a discrete time Markov chain with absorbing state 1,

$$P\{H_n(A) = 1 \mid H_{n-1}(A) = 1\} = 1$$

$$P\{H_n(A) = 1 \mid H_{n-1}(A) = 0\} = P_A \tag{3.4}$$

The firing interval between rounds is independent and exponential with the rate of λ'_A for anti-tank weapons and λ'_T for offensive tanks. These rates include all various

intervals between rounds fired, which are aiming interval, loading interval, converting interval and firing interval. We will assume that the offensive tanks have a constant velocity through the mine field. Hence, the potential offensive distances traveled between rounds fired are also independent and exponential with the rate of λ_A for the anti-tank weapon and λ_T for the offensive tank.

All contestants have unlimited ammunition supplies and begin the duel with loaded weapons. The duel between offense and defense is assumed to start at the time of the first firing of the defender's anti-tank weapon or at the time the first mine is hit, whichever time is smaller. The duel will be ended when either the offensive or the defensive side is killed or an offensive tank successfully crosses the mine field.

It is assumed that the offensive tanks damage category due to anti-tank weapon firing is only killed or not killed. When the offensive tanks damage category due to mine is loss of mobility or completely destroyed, the anti-tank weapon changes its aim objective to the other tank.

Let "S_A" be the offensive tank position when the anti-tank weapon is killed by an offensive tank, let "S_T" be the offensive tank position when an offensive tank is killed by a defensive anti-tank weapon. Since the intervals between firing are independent and exponential and

assumptions (3.3) and (3.4) hold, and the offensive tanks travel at a constant velocity.

$$P\{S_A > x,\ S_T > x\} = e^{-(\lambda_A P_A)x} \cdot e^{-(\lambda_T P_T)x}$$
$$= e^{-(\lambda_A P_A + \lambda_T P_T)x}$$

and

$$P\{S_A < S_T < x\} = \left(1 - e^{-(\lambda_A P_A + \lambda_T P_T)x}\right)\frac{\lambda_T P_T}{\lambda_A P_A + \lambda_T P_T}$$

for the fuel that lasts at least x distance.

IV. DUEL

Two simple models will be analyzed in detail; the first being that there is one offensive tank and one anti-tank weapon of the defender. The duel starts at the time of the first firing of the defensive anti-tank weapon or the time the first mine is hit, whichever time is smaller. The other one is that there are two offensive tanks and one defensive anti-tank weapon. The offensive tanks start across the field at the same time and along different paths. The anti-tank weapon starts to fire as soon as the offensive tanks enter the field.

If there is only one offensive tank in the field, it is easy for the operator of the anti-tank weapon on the defensive side to wait until the offensive tank goes some distance into the field before the anti-tank weapon fires its first shot. If two offensive tanks are to get into the mine field to attack the defensive forces, it is difficult for the operator of the anti-tank weapon to wait until the offensive tanks arrive at predetermined positions before first defensive firing.

In these two models, five parameters which can affect the result and which can be changed in value with training or other remedies, will be concerned. They are:

(1) The fire rate of an offensive tank (λ_T),

(2) The hit probability of an offensive tank (P_T),

(3) The rate of mines in a mine field (R),

(4) The fire rate of a defensive anti-tank weapon (λ_A),

(5) The hit probability of a defensive anti-tank weapon (P_A).

In the remainder of this section, algebraic solutions for the probability that an offensive tank successfully gets through the mine field for the above models, will be given. In the next section, numerical results and investigation for the sensitivity of the five parameters will be provided.

A. MODEL ONE

In this model, there is one offensive tank crossing a preinstalled mine field. The mine field can be either undefended or defended by one anti-tank weapon.

Let D' be the position of the offensive tank in the field when the defensive anti-tank weapon fires its first shot. The position is greater than or equal to "0" and less than the depth of the mine field ($0 \le D' < D$).

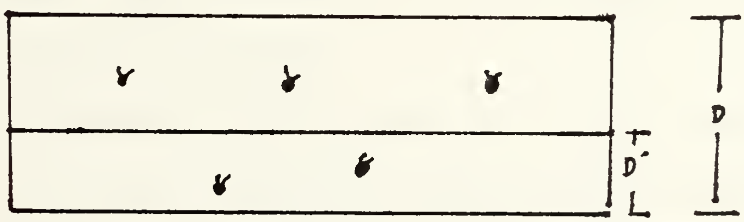

Figure 4. Location of D'

The event that the offensive tank successfully gets through the mine field is the result of several factors; they are, not hit be a mine, hit by the mine but has slight damage; not killed by defender's first shot and not killed by anti-tank weapon in duel during its travel through the remainder of the mine field.

Let

$$N = \begin{cases} 1; & \text{if an offensive tank gets through the mine field successfully.} \\ 0; & \text{otherwise} \end{cases}$$

1. There is No Defensive Anti-Tank Weapon

$$P\{N(D) = 1\} = e^{-RD} + R \cdot D \cdot P_{01} e^{-RD}$$

$$= (1 + R \cdot D \cdot P_{01}) e^{-RD} \qquad (4.1)$$

2. There is One Defensive Anti-Tank Weapon

In this case, we are concerned with the position of the first mine which the offensive tank encounters. If the first mine the offensive tank encounters is less the first defensive firing position (D'), then the potential duel duration is longer than the time for the offensive tank to travel D - D' across the field.

For an offensive tank to get out of the mine field successfully, there is no mine through its path or just one mine the tank encounters, but has slight damage and the

tank is not killed by the anti-tank weapon's first shot, and not killed in the duel. Hence,

$$
\begin{aligned}
P\{N(D)=1\} = {} & e^{-RD}[1-P_A]*\Big[e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')} \\
& \qquad + \left(1-e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')}\right)\frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}\Big] \\
& + e^{-RD'}\left[R(D-D')\,e^{-R(D-D')}P_{01}\right]*[1-P_A] \\
& \quad *\Big[e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')}+\left(1-e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')}\right)\frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}\Big] \\
& + \int_0^{D'}\left[e^{-R(D-x)}\right]*\left[RP_{01}e^{-Rx}\right]*[1-P_A]*\Big[e^{-(\lambda_A P_A+\lambda_T P_T)(D-x)} \\
& \qquad + \left(1-e^{-(\lambda_A P_A+\lambda_T P_T)(D-x)}\right)\frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}\Big]\,dx \\
= {} & e^{-RD}[1-P_A]*\Big[e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')} \\
& \qquad + \left(1-e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')}\right)\frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}\Big] \\
& + e^{-RD'}\left[RP_{01}(D-D')e^{-R(D-D')}\right]*[1-P_A] \\
& \quad *\Big[e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')}+\left(1-e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')}\right)\frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}\Big] \\
& + \frac{RP_{01}e^{-RD}}{\lambda_A P_A+\lambda_T P_T}[1-P_A]*\Big[\left(e^{(\lambda_A P_A+\lambda_T P_T)D'}-1\right)e^{-(\lambda_A P_A+\lambda_T P_T)D} \\
& \qquad \left(1+\frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}\right)+\lambda_T P_T D'\Big]
\end{aligned}
$$

We will now consider the special case in which $P_{ol} = 0$; that is, the offensive tank is killed with probability 1 when it hits a mine. In this case, expression (4.2) becomes

$$P\{N(D)=1\} = e^{-RD}[1-P_A]*[e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')} + \frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}(1-e^{-(\lambda_A P_A+\lambda_T P_T)(D-D')})] \equiv f(D') \tag{4.3}$$

The value of D' that minimizes Equation (4.3) will be found as

$$\frac{\partial}{\partial x}f(x) = e^{-RD}[1-P_A]\,e^{-(\lambda_A P_A+\lambda_T P_T)(D-x)}\,[\lambda_A P_A+\lambda_T P_T] * [1 - \frac{\lambda_T P_T}{\lambda_A P_A+\lambda_T P_T}] \geq 0$$

for $0 \leq x \leq D$.

Hence, for the defense, the optimal strategy is to start firing when the offensive tank first enters the field. It is possible to derive an analytic solution for the optional D' in the case in which $P_{01} > 0$, but it will no doubt be very complicated. In Section V, the optimal distance will be evaluated numerically for various cases of interest.

B. MODEL TWO

In this model there are two offensive tanks to cross the mine field. They start at the other edge of the mine field at the same time and use disjoint paths [Ref. 4].

The mine field can be either undefended or defended by the defender's anti-tank weapon.

Assume that the fire rates and hit probabilities of the offensive tanks are the same and if the mine field is defended, the duel starts at the time when the offensive tanks get into the mind field. The probability that the anti-tank weapon of the defender aims first at any particular offensive tank is ½, whether the first shot of the duel is offensive or defensive is random.

Let $X_1(x)$ be the state of the offensive tank using #1 path x units into the mine field,

$X_2(x)$ be the state of the offensive tank using #2 path x units into the mine field,

$Y(x)$ be the state of the anti-tank weapon when the offensive tank is x units into the mine field. By the previously stated assumptions (3.1), the random variables $X_1(x)$ and $X_2(x)$ can take the value {0, 1, 2, 3} and the random variable $Y(x)$ takes the value $\{\underline{0}, \underline{3}\}$.

1. <u>No Anti-Tank Weapon Defends</u>

We can expect several cases; neither tank encounters a mine in their path; one offensive tank hit a mine and has slight damage or loss of mobility or completely destroyed damage, and the other one has no damage; one offensive tank has slight damage or loss of mobility or completely destroyed damage and the other one has only slight damage.

Hence, letting N(x) be the number of tanks with no damage or only slight damage:

$$
\begin{aligned}
P\{N(D) \geq 1\} = {} & P\{X_1(x) = 0,\ X_2(x) = 0\} \\
& + P\{X_1(x) = 1,\ X_2(x) = 0\} \\
& + P\{X_1(x) = 0,\ X_2(x) = 1\} \\
& + P\{X_1(x) = 2,\ X_2(x) = 0\} \\
& + P\{X_1(x) = 0,\ X_2(x) = 2\} \\
& + P\{X_1(x) = 3,\ X_2(x) = 0\} \\
& + P\{X_1(x) = 0,\ X_2(x) = 3\} \\
& + P\{X_1(x) = 1,\ X_2(x) = 1\} \\
& + P\{X_1(x) = 2,\ X_2(x) = 1\} \\
& + P\{X_1(x) = 1,\ X_2(x) = 2\} \\
& + P\{X_1(x) = 3,\ X_2(x) = 1\} \\
& + P\{X_1(x) = 1,\ X_2(x) = 3\} \qquad (4.5)
\end{aligned}
$$

then

$$
\begin{aligned}
P\{N(D) \geq 1\} = {} & [\mathrm{I}]' + 2[\mathrm{II}]' + 2[\mathrm{III}]' + 2[\mathrm{IV}]' \\
& + [\mathrm{V}]' + 2[\mathrm{VI}]' + 2[\mathrm{VII}]' \qquad (4.6)
\end{aligned}
$$

Here

$$[\mathrm{I}]' = P\{X_1(D)=0,\ X_2(D)=0\} = e^{-RD}\cdot e^{-RD}$$

$$[\mathrm{II}]' = P\{X_1(D)=1,\ X_2(D)=0\} = e^{-RD}(R\cdot P_{01} D e^{-RD})$$

$$= P\{X_1(D)=0,\ X_2(D)=1\}$$

$$[\mathrm{III}]' = P\{X_1(D)=2,\ X_2(D)=0\} = P_{02}(1-e^{-RD})\, e^{-RD}$$

$$= P\{X_1(D)=0,\ X_2(D)=2\}$$

$$[\mathrm{IV}]' = P\{X_1(D)=3,\ X_2(D)=0\} = P_{03}(1-e^{-RD})\, e^{-RD}$$

$$+ P_{01}P_{13}(1-e^{-RD}-RDe^{-RD})\, e^{-RD}$$

$$= P\{X_1(D)=0,\ X_2(D)=3\}$$

$$[\mathrm{V}]' = P\{X_1(D)=1,\ X_2(D)=1\} = (RP_{01} D e^{-RD})^2$$

$$[\mathrm{VI}]' = P\{X_1(D)=2,\ X_2(D)=1\} = (RP_{01} D e^{-RD})\cdot P_{02}(1-e^{-RD})$$

$$= P\{X_1(D)=1,\ X_2(D)=2\}$$

$$[\mathrm{VII}]' = P\{X_1(D)=3,\ X_2(D)=1\} = (RP_{01} D e^{-RD}) * [P_{03}(1-e^{-RD})$$

$$+ P_{01}P_{13}(1-e^{-RD}-RDe^{-RD})]$$

$$= P\{X_1(D)=1,\ X_2(D)=3\}$$

2. <u>There is One Defensive Anti-Tank Weapon</u>

This case is more complicated than the first case. Again, let N(x) be the number of tanks with no damage or only slight damage.

$$
\begin{aligned}
P\{N(D) \geq 1\} = \; & P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\} \\
& + P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\} \\
& + P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=0\} \\
& + P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=2\} \\
& + P\{Y(x)=0,\ X_1(x)=2,\ X_2(x)=0\} \\
& + P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=3\} \\
& + P\{Y(x)=0,\ X_1(x)=3,\ X_2(x)=0\} \\
& + P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=1\} \\
& + P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=2\} \\
& + P\{Y(x)=0,\ X_1(x)=2,\ X_2(x)=1\} \\
& + P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=3\} \\
& + P\{Y(x)=0,\ X_1(x)=3,\ X_2(x)=1\} \\
& + P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\} \\
& + P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=1\} \\
& + P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=0\} \\
& + P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=2\} \\
& + P\{Y(x)=3,\ X_1(x)=2,\ X_2(x)=0\} \\
& + P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=3\} \\
& + P\{Y(x)=3,\ X_1(x)=3,\ X_2(x)=0\}
\end{aligned}
\tag{4.7}
$$

$$+ P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=1\}$$
$$+ P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=2\}$$
$$+ P\{Y(x)=3,\ X_1(x)=2,\ X_2(x)=1\}$$
$$+ P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=3\}$$
$$+ P\{Y(x)=3,\ X_1(x)=3,\ X_2(x)=1\}$$

Finally,

$$\begin{aligned} P\{N(D)\geq 1\} = [I] + 2[II] + 2[III] + 2[IV] + [V] + 2[VI] + 2[VII] \\ + [VIII] + 2[IX] + 2[X] + 2[XI] + [XII] + 2[XIII] + 2[XIV] \end{aligned} \tag{4.8}$$

where [1], [II], ..., [XIV] are defined as below.

$$[I] = P\{Y(D)=0,\ X_1(D)=0,\ X_2(D)=0\}$$

By the exponential assumptions of the model, for h small

$$\begin{aligned} &P\{Y(x+h)=0,\ X_1(x+h)=0,\ X_2(x+h)=0\} \\ &= P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\}\,[1-2Rh+o(h)] \\ &\qquad * [1-(\lambda_A P_A + 2\lambda_T P_T)h + o(h)] \end{aligned}$$

Subtracting $P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\}$ from both sides, dividing by h and letting h tend to zero, results in

$$\begin{aligned} &\frac{d}{dx} P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\} \\ &= -(2R+\lambda_A P_A + 2\lambda_T P_T) * P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\} \end{aligned}$$

Solving

$$P\{Y(D)=0,\ X_1(D)=0,\ X_2(x)=0\}$$
$$= e^{-(2R+\lambda_A P_A+2\lambda_T P_T)D}.$$

$$[II] = P\{Y(D)=0,\ X_1(D)=0,\ X_2(D)=1\}$$

Similarly,

$$P\{Y(x+h)=0,\ X_1(x+h)=0,\ X_2(x+h)=1\}$$
$$= P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\}[1-2Rh+o(h)]$$
$$* [1-(\lambda_A P_A+2\lambda_T P_T)h+o(h)]$$
$$+ P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\}[Rh+o(h)]P_{01}$$

$$\frac{d}{dx}P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\}$$
$$= -(2R+\lambda_A P_A+2\lambda_T P_T)*P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\}$$
$$+ (RP_{01})\,e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}$$

then

$$P\{Y(D)=0,\ X_1(D)=0,\ X_2(D)=1\}$$
$$= R\cdot P_{01}\cdot D\cdot e^{-(2R+\lambda_A P_A+2\lambda_T P_T)D}$$
$$= P\{Y(D)=0,\ X_1(D)=1,\ X_2(D)=0\}$$

similarly,

$$[\text{III}] = P\{Y(D)=0,\ X_1(D)=0,\ X_2(D)=2\}$$

$$P\{Y(x+h)=0,\ X_1(x+h)=0,\ X_2(x+h)=2\}$$

$$= P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=2\}[1-(R+\lambda_A P_A + 2\lambda_T P_T)h + o(h)]$$

$$+ P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\}[RP_{02}h + o(h)]$$

Then using the expression in [I]

$$\frac{d}{dx}P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=2\}$$

$$= -(R+\lambda_A P_A + 2\lambda_T P_T) * P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=2\}$$

$$+ (RP_{02})\, e^{-(2R+\lambda_A P_A + 2\lambda_T P_T)x}$$

$$P\{Y(D)=0,\ X_1(D)=0,\ X_2(D)=2\}$$

$$= e^{-RD}(1-e^{-RD})P_{02}\, e^{-(\lambda_A P_A + 2\lambda_T P_T)D}$$

$$= P\{Y(D)=0,\ X_1(D)=2,\ X_2(D)=0\}$$

$$[\text{IV}] = P\{Y(D)=0,\ X_1(D)=0,\ X_2(D)=3\}$$

$$P\{Y(x+h)=0,\ X_1(x+h)=0,\ X_2(x+h)=3\}$$

$$= P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=3\}[1-(R+\lambda_A P_A + \lambda_T P_T)h + o(h)]$$

$$+ P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\}[RP_{03}h + \tfrac{1}{2}\lambda_A P_A h + o(h)]$$

$$+ P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\}[RP_{13}h + \tfrac{1}{2}\lambda_A P_A h + o(h)]$$

$$\frac{d}{dx} P\{Y(x)=0,\quad X_1(x)=0,\quad X_2(x)=3\}$$

$$= -(R+\lambda_A P_A+\lambda_T P_T) * P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=3\}$$

$$+ (R P_{03} + \tfrac{1}{2}\lambda_A P_A)\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}$$

$$+ (R P_{13} + \tfrac{1}{2}\lambda_A P_A) * [R P_{01}\, x\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}]$$

$$P\{Y(D)=0,\quad X_1(D)=0,\quad X_2(D)=3\}$$

$$= e^{-RD} \cdot R\cdot P_{03} \frac{1}{R+\lambda_T P_T}\left[e^{-(\lambda_A P_A+\lambda_T P_T)D} - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right]$$

$$+ e^{-RD} R^2 P_{01} P_{13} \frac{1}{(R+\lambda_T P_T)^2}\left[e^{-(\lambda_A P_A+\lambda_T P_T)D} - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right.$$

$$\left. -(R+\lambda_T P_T)\cdot D\cdot e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right]$$

$$+ e^{-RD} \cdot \frac{\lambda_A P_A}{2(R+\lambda_T P_T)^2}\left[e^{-(\lambda_A P_A+\lambda_T P_T)D} - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right]$$

$$+ e^{-RD} \cdot R\cdot P_{01} \frac{\lambda_A P_A}{2(R+\lambda_T P_T)^2}\left[e^{-(\lambda_A P_A+\lambda_T P_T)D} - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right.$$

$$\left. -(R+\lambda_T P_T)\cdot D\cdot e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right]$$

$$= P\{Y(D)=0,\ X_1(D)=3,\ X_2(D)=0\}$$

$$[\text{V}] = P\{Y(D)=0,\ X_1(D)=1,\ X_2(D)=1\}$$

$$P\{Y(x+h)=0,\ X_1(x+h)=1,\ X_2(x+h)=1\}$$

$$= P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=1\}[1-(2R+\lambda_A P_A+2\lambda_T P_T)h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=0\}[RP_{01}h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\}[RP_{01}h+o(h)]$$

$$\frac{d}{dx}P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=1\}$$

$$= -(2R+\lambda_A P_A+2\lambda_T P_T)*P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=1\}$$

$$+(2RP_{01})[RP_{01}\,x\,e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}]$$

$$P\{Y(D)=0,\ X_1(D)=1,\ X_2(D)=1\}$$

$$= (R\cdot P_{01}\cdot D\cdot e^{-RD})^2 e^{-(\lambda_A P_A+2\lambda_T P_T)D}$$

$$[\text{VI}] = P\{Y(D)=0,\ X_1(D)=1,\ X_2(D)=2\}$$

$$P\{Y(x+h)=0,\ X_1(x+h)=1,\ X_2(x+h)=2\}$$

$$= P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=2\}[1-(R+\lambda_A P_A+2\lambda_T P_T)h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=0\}[RP_{02}h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=2\}[RP_{01}h+o(h)]$$

$$\frac{d}{dx} P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=2 \}$$

$$= -(R+\lambda_A P_A + 2\lambda_T P_T) * P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=2 \}$$

$$+ (R P_{02}) [R P_{01} x\, e^{-(2R+\lambda_A P_A + 2\lambda_T P_T)x}]$$

$$+ (R P_{01}) [e^{-Rx} (1-e^{-Rx}) P_{02}\, e^{-(\lambda_A P_A + 2\lambda_T P_T)x}]$$

$$P\{ Y(D)=0,\ X_1(D)=1,\ X_2(D)=2 \}$$

$$= R \cdot D \cdot P_{01} e^{-RD} (1-e^{-RD}) P_{02}\, e^{-(\lambda_A P_A + 2\lambda_T P_T)D}$$

$$= P\{ Y(D)=0,\ X_1(D)=2,\ X_2(D)=1 \}$$

$$[\mathrm{VII}] = P\{ Y(D)=0,\ X_1(D)=1,\ X_2(D)=3 \}$$

$$P\{ Y(x+h)=0,\ X_1(x+h)=1,\ X_2(x+h)=3 \}$$

$$= P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=3 \} [1-(R+\lambda_A P_A + \lambda_T P_T)h + o(h)]$$

$$+ P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=0 \} [R P_{03} h + \tfrac{1}{2}\lambda_A P_A h + o(h)]$$

$$+ P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=1 \} [R P_{13} h + \tfrac{1}{2}\lambda_A P_A h + o(h)]$$

$$+ P\{ Y(x)=0,\ X_1(x)=0,\ X_2(x)=3 \} [R P_{01} h + o(h)]$$

$$\frac{d}{dx} P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=3 \}$$

$$= -(R+\lambda_A P_A + \lambda_T P_T) * P\{ Y(x)=0,\ X_1(x)=1,\ X_2(x)=3 \}$$

$$+(RP_{03}+\tfrac{1}{2}\lambda_A P_A)\left[R\cdot x\cdot P_{01}\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}\right]$$

$$+(RP_{13}+\tfrac{1}{2}\lambda_A P_A)\left[R^2 x^2 P_{01}^2\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}\right]$$

$$+(RP_{01})\left[\left(\frac{RP_{03}}{R+\lambda_T P_T}+\frac{R^2 P_{01}P_{13}}{(R+\lambda_T P_T)^2}\right)e^{-(R+\lambda_A P_A+\lambda_T P_T)x}\right]$$

$$+\left(\frac{\lambda_A P_A}{2(R+\lambda_T P_T)}+\frac{RP_{01}\lambda_A P_A}{2(R+\lambda_T P_T)^2}\right)e^{-(R+\lambda_A P_A+\lambda_T P_T)x}$$

$$-\left(\frac{RP_{03}}{R+\lambda_T P_T}+\frac{R^2 P_{01}P_{13}}{(R+\lambda_T P_T)^2}\right)e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}$$

$$-\left(\frac{\lambda_A P_A}{2(R+\lambda_T P_T)}+\frac{RP_{01}\lambda_A P_A}{2(R+\lambda_T P_T)^2}\right)e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}$$

$$-\left(\frac{R^2 P_{01}P_{13}}{R+\lambda_T P_T}\right)x\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}$$

$$-\left(\frac{RP_{01}\lambda_A P_A}{2(R+\lambda_T P_T)}\right)x\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}\ \Big]$$

$$P\{Y(D)=0,\quad X_1(D)=1,\quad X_2(D)=3\}$$

$$=RP_{01}De^{-RD}\cdot R\frac{1}{(R+\lambda_T P_T)}\left[e^{-(\lambda_A P_A+\lambda_T P_T)D}-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right]$$

$$+RP_{01}De^{-RD}R^2P_{01}P_{13}\frac{1}{(R+\lambda_T P_T)^2}\Big[e^{-(\lambda_A P_A+\lambda_T P_T)D}-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}$$

$$-(R+\lambda_T P_T)D\, e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\Big]$$

$$+RP_{01}De^{-RD}\frac{\lambda_A P_A}{2(R+\lambda_T P_T)}\left[e^{-(\lambda_A P_A+\lambda_T P_T)D}-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right]$$

$$+RP_{01}De^{-RD}RP_{01}\frac{\lambda_A P_A}{2(R+\lambda_T P_T)^2}\Big[e^{-(\lambda_A P_A+\lambda_T P_T)D}-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}$$

$$-(R+\lambda_T P_T)D\, e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\Big]$$

$$=P\{Y(D)=0,\quad X_1(D)=3,\quad X_2(D)=1\}$$

$$[\mathrm{VIII}] = P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=0\}$$

$$P\{Y(x+h)=3,\ X_1(x+h)=0,\ X_2(x+h)=0\}$$

$$= P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\}[1-2Rh+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=0\}[2\lambda_T P_T h+o(h)]$$

$$\frac{d}{dx}P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\}$$

$$= -(2R)*P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\}$$

$$+(2\lambda_T P_T)\,e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}$$

$$P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=0\}$$

$$= \frac{2\lambda_T P_T}{\lambda_A P_A+2\lambda_T P_T}\,e^{-2RD}\left(1-e^{-(\lambda_A P_A+2\lambda_T P_T)D}\right)$$

$$[\mathrm{IX}] = P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=1\}$$

$$P\{Y(x+h)=3,\ X_1(x+h)=0,\ X_2(x+h)=1\}$$

$$= P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=1\}[1-2Rh+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\}[RP_{01}h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=1\}[2\lambda_T P_T h+o(h)]$$

$$\frac{d}{dx} P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=1\}$$

$$= -(2R) * P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=1\}$$

$$+ (RP_{01})\left[\frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}\left(e^{-2Rx} - e^{-(2R+\lambda_A P_A + 2\lambda_T P_T)x}\right)\right]$$

$$+ (2\lambda_T P_T)\left[RP_{01}\, x\, e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}\right]$$

$$P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=1\}$$

$$= e^{-RD} R D P_{01} e^{-RD}\left(1 - e^{-(\lambda_A P_A + 2\lambda_T P_T)D}\right)\frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}$$

$$= P\{Y(D)=3,\ X_1(D)=1,\ X_2(D)=0\}$$

$$[\text{X}] = P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=2\}$$

$$P\{Y(x+h)=3,\ X_1(x+h)=0,\ X_2(x+h)=2\}$$

$$= P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=2\}[1 - Rh + o(h)]$$

$$+ P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\}[RP_{02}h + o(h)]$$

$$+ P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=2\}[2\lambda_T P_T h + o(h)]$$

$$\frac{d}{dx}P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=2\}$$

$$= -R * P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=2\}$$

$$+(RP_{02})\left[\frac{2\lambda_T P_T}{\lambda_A P_A+2\lambda_T P_T}e^{-2Rx}-\frac{2\lambda_T P_T}{\lambda_A P_A+2\lambda_T P_T}e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}\right]$$

$$+(2\lambda_T P_T)\left[P_{02}e^{-(R+\lambda_A P_A+2\lambda_T P_T)x}-P_{02}e^{-(2R+\lambda_A P_A+2\lambda_T P_T)x}\right]$$

$$P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=2\}$$

$$= e^{-RD}(1-e^{-RD})P_{02}\left(1-e^{-(\lambda_A P_A+2\lambda_T P_T)D}\right)\frac{2\lambda_T P_T}{\lambda_A P_A+2\lambda_T P_T}$$

$$= P\{Y(D)=3,\ X_1(D)=2,\ X_2(D)=0\}$$

$$[XI] = P\{Y(D)=3,\ X_1(D)=0,\ X_2(x)=3\}$$

$$P\{Y(x+h)=3,\ X_1(x+h)=0,\ X_2(x+h)=3\}$$

$$= P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=3\}[1-Rh+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=0\}[RP_{03}h+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=1\}[RP_{13}h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=0,\ X_2(x)=3\}[\lambda_T P_T h+o(h)]$$

$$\frac{d}{dx} P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=3\}$$

$$= -R * P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=3\}$$

$$+ (R P_{03})\left[\frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T} e^{-2Rx}\left(1 - e^{-(\lambda_A P_A + 2\lambda_T P_T)x}\right)\right]$$

$$+ (R P_{13})\left[e^{-Rx} R x P_{01} e^{-Rx}\left(1 - e^{-(\lambda_A P_A + 2\lambda_T P_T)x}\right)\frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}\right]$$

$$+ (\lambda_T P_T)\left[e^{-Rx} R P_{03} \frac{1}{R+\lambda_T P_T}\left(e^{-(\lambda_A P_A + \lambda_T P_T)x} - e^{-(R+\lambda_A P_A + 2\lambda_T P_T)x}\right)\right.$$

$$+ e^{-Rx} R P_{03} \frac{1}{(R+\lambda_T P_T)^2}\left(e^{-(\lambda_A P_A + \lambda_T P_T)x} - e^{-(R+\lambda_A P_A + 2\lambda_T P_T)x}\right.$$

$$\left. - (R+\lambda_T P_T)x\, e^{-(R+\lambda_A P_A + 2\lambda_T P_T)x}\right)$$

$$+ e^{-Rx} \frac{\lambda_A P_A}{2(R+\lambda_T P_T)^2}\left(e^{-(\lambda_A P_A + \lambda_T P_T)x} - e^{-(R+\lambda_A P_A + 2\lambda_T P_T)x}\right)$$

$$+ e^{-Rx} R P_{01} \frac{\lambda_A P_A}{2(R+\lambda_T P_T)^2}\left(e^{-(\lambda_A P_A + \lambda_T P_T)x} - e^{-(R+\lambda_A P_A + 2\lambda_T P_T)x}\right.$$

$$\left.\left. - (R+\lambda_T P_T)x\, e^{-(R+\lambda_A P_A + 2\lambda_T P_T)x}\right)\right]$$

$$P\{Y(D)=3,\ X_1(D)=0,\ X_2(D)=3\}$$

$$= e^{-RD}\left[\frac{\lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_T P_T)(\lambda_A P_A + \lambda_T P_T)}\left(1 - e^{-(\lambda_A P_A + \lambda_T P_T)D}\right)\right.$$

$$+\frac{\lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_T P_T)(\lambda_A P_A+\lambda_T P_T)}\left(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right)\Big]$$

$$+e^{-RD} R P_{01}\Big[\frac{\lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_A P_A)^2(R+\lambda_T P_T)}\left(1-e^{-(\lambda_A P_A+\lambda_T P_T)D}\right)$$

$$-\frac{\lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_T P_T)^2(R+\lambda_A P_A+2\lambda_T P_T)}\left(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right)$$

$$-\frac{\lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_A P_A)(R+\lambda_A P_A+2\lambda_T P_T)^2}\left(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right)$$

$$+\frac{\lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_A P_A)(R+\lambda_A P_A+2\lambda_T P_T)} D e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\Big]$$

$$+e^{-RD} P_{03}\Big[\frac{2\lambda_T P_T}{\lambda_A P_A+2\lambda_T P_T}\left(1-e^{-RD}\right)$$

$$+\frac{R\cdot \lambda_T P_T}{(R+\lambda_T P_T)(\lambda_A P_A+\lambda_T P_T)}\left(1-e^{-(\lambda_A P_A+\lambda_T P_T)D}\right)$$

$$-\frac{2R\cdot \lambda_T P_T}{(R+\lambda_T P_T)(R+\lambda_A P_A+2\lambda_T P_T)}\left(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right)\Big]$$

$$+e^{-RD} P_{01} P_{13}\Big[\frac{R^2 \lambda_T P_T}{(R+\lambda_T P_T)^2(\lambda_A P_A+\lambda_T P_T)}\left(1-e^{-(\lambda_A P_A+\lambda_T P_T)D}\right)$$

$$-\frac{R^2 \lambda_T P_T}{(R+\lambda_T P_T)(R+\lambda_A P_A+2\lambda_T P_T)^2}\left(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\right)$$

$$-\frac{R^2 \lambda_T P_T}{(R+\lambda_T P_T)^2(R+\lambda_A P_A+2\lambda_T P_T)^2}\Big(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}$$

$$-(R+\lambda_T P_T) D e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\Big)$$

$$- \frac{2R^2 \cdot \lambda_T P_T}{(\lambda_A P_A + 2\lambda_T P_T)(R + \lambda_A P_A + 2\lambda_T P_T)^2} \left(1 - e^{-(R + \lambda_A P_A + 2\lambda_T P_T)D}\right)$$

$$- (R + \lambda_A P_A + 2\lambda_T P_T) D e^{-(R + \lambda_A P_A + 2\lambda_T P_T)D}$$

$$+ \frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T} \left(1 - e^{-RD} - RD e^{-RD}\right)]$$

$$= P\{ Y(D) = 3, \ X_1(D) = 3, \ X_2(D) = 0 \}$$

$$[XII] = P\{ Y(D) = 3, \ X_1(D) = 1, \ X_2(D) = 1 \}$$

$$P\{ Y(x+h) = 3, \ X_1(x+h) = 1, \ X_2(x+h) = 1 \}$$

$$= P\{ Y(x) = 3, \ X_1(x) = 1, \ X_2(x) = 1 \} [1 - 2Rh + o(h)]$$

$$+ P\{ Y(x) = 3, \ X_1(x) = 1, \ X_2(x) = 0 \} [RP_{01} h + o(h)]$$

$$+ P\{ Y(x) = 3, \ X_1(x) = 0, \ X_2(x) = 1 \} [RP_{01} h + o(h)]$$

$$+ P\{ Y(x) = 0, \ X_1(x) = 1, \ X_2(x) = 1 \} [2\lambda_T P_T h + o(h)]$$

$$\frac{d}{dx} P\{ Y(x) = 3, \ X_1(x) = 1, \ X_2(x) = 1 \}$$

$$= -2R * P\{ Y(x) = 3, \ X_1(x) = 1, \ X_2(x) = 1 \}$$

$$+(2RP_{01})\left[\frac{RP_{01}\cdot 2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}\,x\,\left(e^{-2Rx} - e^{-(2R+\lambda_A P_A + 2\lambda_T P_T)x}\right)\right]$$

$$+(2\lambda_T P_T)\left[R^2 P_{01}^2 x^2 e^{-(2R+\lambda_A P_A + 2\lambda_T P_T)x}\right]$$

$$P\{Y(D)=3,\ X_1(D)=1,\ X_2(D)=1\}$$

$$= RDP_{01}e^{-RD}\cdot RDP_{01}e^{-RD}\left(1-e^{-(\lambda_A P_A + 2\lambda_T P_T)}\right)\frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}\,.$$

$$[\mathrm{XIII}] = P\{Y(D)=3,\ X_1(D)=1,\ X_2(D)=2\}$$

$$P\{Y(x+h)=3,\ X_1(x+h)=1,\ X_2(x+h)=2\}$$

$$= P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=2\}[1-Rh+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=0\}[RP_{02}h+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=2\}[RP_{01}h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=2\}[2\lambda_T P_T h+o(h)]$$

$$\frac{d}{dx}P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=2\}$$

$$= -R * P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=2\}$$

$$+(RP_{02})\left[e^{-Rx} R x P_{01} e^{-Rx}(1-e^{-(\lambda_A P_A+2\lambda_T P_T)x}) \frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}\right]$$

$$+(RP_{01})\left[e^{-Rx}(1-e^{-Rx}) P_{02}(1-e^{-(\lambda_A P_A+2\lambda_T P_T)x}) \frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}\right]$$

$$+(2\lambda_T P_T)\left[R x e^{-Rx} P_{01}(1-e^{-Rx}) P_{02}\, e^{-(\lambda_A P_A+2\lambda_T P_T)x}\right]$$

$$P\{Y(D)=3,\ X_1(D)=1,\ X_2(D)=2\}$$

$$= R D P_{01} e^{-RD}(1-e^{-RD}) P_{02}(1-e^{-(\lambda_A P_A+2\lambda_T P_T)D}) \frac{2\lambda_T P_T}{\lambda_A P_A + 2\lambda_T P_T}$$

$$= P\{Y(D)=3,\ X_1(D)=2,\ X_2(D)=1\}$$

$$[XIV] = P\{Y(D)=3,\ X_1(D)=1,\ X_2(D)=3\}$$

$$P\{Y(x+h)=3,\ X_1(x+h)=1,\ X_2(x+h)=3\}$$

$$= P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=3\}[1-Rh+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=0\}[RP_{03}h+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=1,\ X_2(x)=1\}[RP_{13}h+o(h)]$$

$$+P\{Y(x)=3,\ X_1(x)=0,\ X_2(x)=3\}[RP_{01}h+o(h)]$$

$$+P\{Y(x)=0,\ X_1(x)=1,\ X_2(x)=3\}[\lambda_T P_T h+o(h)]$$

$$P\{Y(D)=3,\ X_1(D)=1,\ X_2(D)=3\} = \alpha_1+\alpha_2+\alpha_3+\alpha_4$$

$$=P\{Y(D)=3,\ X_1(D)=3,\ X_2(D)=1\}$$

here

$$\alpha_1 = e^{-RD}RP_{01}\Big\{\frac{\lambda_T P_T}{2(R+\lambda_T P_T)(\lambda_A P_A+\lambda_T P_T)}\Big[\frac{2R}{(\lambda_A P_A+\lambda_T P_T)}\big(1-e^{-(\lambda_A P_A+\lambda_T P_T)D}\big)$$

$$+\lambda_A P_A\cdot D\big(1-e^{-(\lambda_A P_A+\lambda_T P_T)D}\big)$$

$$-\big(R\cdot D\cdot e^{-(\lambda_A P_A+\lambda_T P_T)D}\big)\Big]$$

$$-\frac{\lambda_T P_T}{2(R+\lambda_T P_T)(R+\lambda_A P_A+2\lambda_T P_T)}\Big[\frac{2R}{R+\lambda_A P_A+2\lambda_T P_T}\big(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}$$

$$+(R+\lambda_A P_A+2\lambda_T P_T)De^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\big)$$

$$-\lambda_A P_A\cdot D\cdot\big(1-e^{-(R+\lambda_A P_A+2\lambda_T P_T)D}\big)\Big]\Big\}$$

$$
\begin{aligned}
\alpha_2 = e^{-RD} R P_{01} \Bigg\{ & \frac{R P_{01} \cdot \lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_T P_T)(R+\lambda_A P_A+2\lambda_T P_T)} \Bigg[\frac{1}{(R+\lambda_A P_A)} \Big(1 - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \\
& \qquad - D \Big) \\
& \qquad + \frac{2}{(R+\lambda_A P_A+2\lambda_T P_T)} \Big(1 - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \\
& \qquad - D \Big) \\
& \qquad - D e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \Bigg] \\
& + \frac{R P_{01} \cdot \lambda_A P_A \, \lambda_T P_T}{2(R+\lambda_A P_A)^2 (\lambda_A P_A+\lambda_T P_T)} D \cdot \Big(1 - e^{-(\lambda_A P_A+\lambda_T P_T)D} \Big) \\
& - \frac{R P_{01} \cdot \lambda_A P_A \cdot \lambda_T P_T}{2(R+\lambda_A P_A)^2 (R+\lambda_A P_A+2\lambda_T P_T)} \Bigg[\frac{2}{(R+\lambda_A P_A+2\lambda_T P_T)^2} \Big(1 - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \Big) \\
& \qquad + \frac{1}{(R+\lambda_A P_A+2\lambda_T P_T)} \Big(1 - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \\
& \qquad - 2D e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \Big) \\
& \qquad + D e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} (1-D) \Bigg] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_3 = e^{-RD} R P_{01} \Bigg\{ & \frac{2 P_{03} \cdot \lambda_T P_T}{(\lambda_A P_A+2\lambda_T P_T)} D \left(1 - e^{-RD} \right) \\
& + \frac{2 P_{03} R \lambda_T P_T}{(\lambda_A P_A+2\lambda_T P_T)(R+\lambda_A P_A+2\lambda_T P_T)^2} D \Big[1 - e^{-(R+\lambda_A P_A+2\lambda_T P_T)D} \\
& \qquad - (R+\lambda_A P_A+2\lambda_T P_T)D \Big] \\
& + \frac{R P_{03} \lambda_T P_T}{(R+\lambda_T P_T)(\lambda_A P_A+\lambda_T P_T)^2} \Big[1 - e^{-(\lambda_A P_A+\lambda_T P_T)D} \\
& \qquad - (\lambda_A P_A+\lambda_T P_T)D \Big] \\
& + \frac{2 R P_{03} \lambda_T P_T}{(R+\lambda_T P_T)(R+\lambda_A P_A+2\lambda_T P_T)^2} \Big[1 - e^{-(R-\lambda_A P_A+2\lambda_T P_T)D} - (R+\lambda_A P_A+2\lambda_T P_T)D \Big] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_4 = e^{-RD} R P_{01} \Bigg\{ & \frac{2 P_{01} P_{13} \lambda_T P_T}{(\lambda_A P_A + \lambda_T P_T)} D \left(1 - e^{-RD} - RD e^{-RD}\right) \\
& + \frac{R^2 P_{01} P_{13} \lambda_T P_T}{(R+\lambda_T P_T)^2 (\lambda_A P_A + \lambda_T P_T)} D \left(1 - D e^{-(\lambda_A P_A + \lambda_T P_T) D}\right) \\
& + \frac{2 R^2 P_{01} P_{13} \lambda_T P_T}{(R+\lambda_T P_T)(R+\lambda_A P_A + 2\lambda_T P_T)^3} \Big[1 - e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \\
& \qquad - (R+\lambda_A P_A + 2\lambda_T P_T) D e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \\
& \qquad + (R+\lambda_A P_A + 2\lambda_T P_T) D - \frac{R+\lambda_A P_A + 2\lambda_T P_T}{R+\lambda_T P_T} D \Big] \\
& - \frac{2 R^2 P_{01} P_{13} \lambda_T P_T}{(R+\lambda_T P_T)^2 (R+\lambda_A P_A + 2\lambda_T P_T)^3} \Big[1 - e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \\
& \qquad - (R+\lambda_A P_A + 2\lambda_T P_T) D e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \\
& \qquad - (R+\lambda_A P_A + 2\lambda_T P_T)^2 D e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \\
& \qquad - (R+\lambda_A P_A + 2\lambda_T P_T)^2 D^2 e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \Big] \\
& - \frac{2 R^2 P_{01} P_{13} \lambda_T P_T}{(\lambda_A P_A + 2\lambda_T P_T)(R+\lambda_A P_A + 2\lambda_T P_T)^2} D \Big[1 - D e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \\
& \qquad - (R+\lambda_A P_A + 2\lambda_T P_T) D^2 e^{-(R+\lambda_A P_A + 2\lambda_T P_T) D} \Big] \Bigg\}
\end{aligned}
$$

V. NUMERICAL RESULTS

In this section, numerical results are presented for each model with specific parameter values. Insofar as is possible, the parameter values that are encountered in the field will be used.

A. MODEL ONE

The numerical results for Model One will be given for the following basic parameter values:

$$P_{01} = 0.2 \qquad \lambda_A = \lambda_T = 0.037$$

$$P_{02} = 0.3 \qquad R = 0.005 \qquad (5.1)$$

$$P_{03} = 0.5 \qquad P_A = P_T = 0.6$$

Here all parameter rates are converted into units of distance; for example, the value of the fire rate (λ_A, λ_T) is 0.037 rounds per unit distance which is equal to 5 rounds fired per minute times the speed of an offensive tank in the mine field which is 8/60 kilometers per minute.

1. There is No Defensive Anti-Tank Weapon

Table I gives the P{N(D)=1} for some values of D, using (4.1) and (5.1).

TABLE I

Probability that the Tank Gets
Through the Mine Field Successfully

D	P{N(D) = 1}
300	0.2901
400	0.1895
500	0.1231

D, depth of a mine field in meters.

If the defender wants to reinforce a mine field with no anti-tank weapon defense, there is no effective difference between increasing the rate of mines or lengthening the depth of the mine field as can be seen from Equation (4.1).

2. <u>One Anti-Tank Weapon Defends</u>

First of all, the optimal offensive tank position for the first shot of the defensive anti-tank weapon is evaluated.

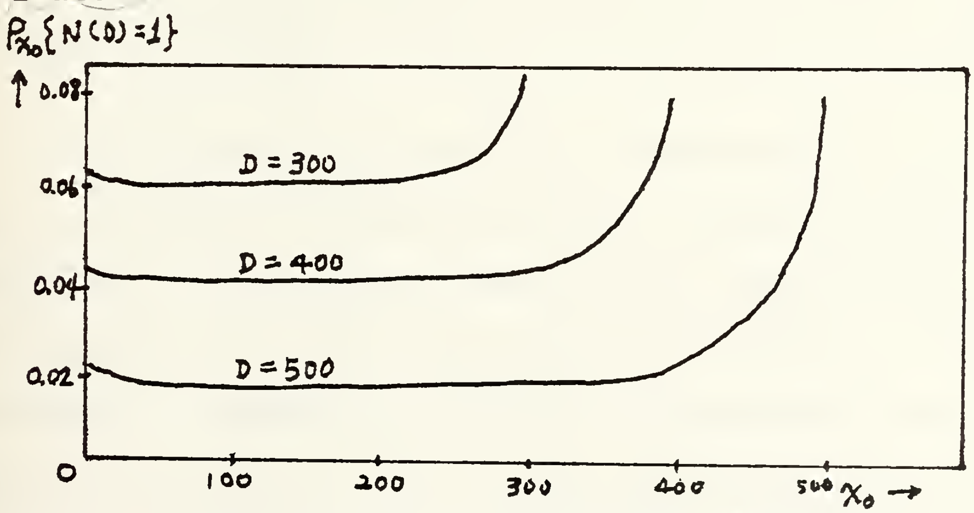

Figure 5. Probability of an Offensive Tank Which Gets Through a Mine Field Successfully.

Speed of offensive tank; 8 kilometers per hour.

Figure 5 presents typical graphs obtained from the numerical evaluation of equation (4.2) for $P_{x_o}\{N(D)=1\}$, the probability the tank successfully travels through the mine field in the case that the anti-tank weapon fires first when the offensive tank is at position X_0.

Optimal distances for the first shot of a defensive anti-tank weapon are given in Table II for the parameter values given in (5.1) for three mine field depths.

TABLE II

Probability of an Offensive Tank
Which Gets Through the Mine Field Successfully

Depth in Meters	P{N(D) = 1}			
	Maximum		Minimum	
	Probability	Position	Probability	Position
300	0.1036	300	0.058	3
400	0.0656	400	0.037	21
500	0.0414	500	0.024	47

The optimal strategy for the defense is to fire its first show at the offensive position which minimizes the P{N(D) = 1}. The optimal position for the defense is changed by changes in the mine field depth and changes in the basic parameters. However, the change in the optimal position appears to be small for the range of parameters we are

interested in. Table III gives the percent change in $P\{N(D) = 1\}$ when parameter values are changed for a field depth of 300 meters and a defensive first shot position of three meters.

%	P_T	λ_T	R	P_A	λ_A
10	4.76	4.76	-11.94	-19.05	-4.76
20	9.09	9.09	-22.50	-36.36	-9.09
30	13.04	13.04	-31.82	-52.17	-13.04
40	16.67	16.67	-40.05	-66.67	-16.67
50	20.00	20.00	-47.31	-80.00	-20.00

TABLE III

Change of Percent of Probability with Advantage of Defender Side

The hit probability of offensive tank (P_T); 0.6

The fire rate of offensive tank (λ_T); 0.037

The rate of mines in the field (R); 0.005

The hit probability of anti-tank weapon (P_A); 0.6

The fire rate of anti-tank weapon (λ_A); 0.037

The value in a cell of Table III indicates the change in percent of initial probability that an offensive tank gets through the mine field due to the amount of change in the parameter values. A positive value means an increased $P\{N(D) = 1\}$ and a negative value is the opposite. Table III suggests that in the case of a duel between one offensive

tank and one defensive anti-tank weapon with mine field, the most sensitive parameter is the hit probability of the defensive anti-tank weapon and the next sensitive is the rate of the mine field. The reason the hit probability of the anti-tank weapon is the most sensitive parameter is that the anti-tank weapon of the defensive forces has the advantage of firing its first shot at an offensive tank by surprise. Table IV gives the same information as Table III for the case in which the anti-tank weapon does not have this advantage; that is, the first shot of the duel is offensive or defensive with a probability of ½.

TABLE IV

Change of Percent of Probability
Without an Advantage of Defender Side

%	P_T	λ_T	R	P_A	λ_A
10	4.76	4.76	-11.94	- 4.76	- 4.76
20	9.09	9.09	-22.50	- 9.09	- 9.09
30	13.04	13.04	-31.82	-13.04	-13.04
40	16.67	16.67	-40.05	-16.67	-16.67
50	20.00	20.00	-47.31	-20.00	-20.00

B. MODEL TWO

The numerical results for Model Two will use the same basic parameter values as Model One; that is:

$P_{01} = 0.2$ $\lambda_T = 0.037$

$P_{02} = 0.3$ $\lambda_A = 0.037$

$P_{03} = 0.5$ $R = 0.005$

$P_T = 0.6$ $P_A = 0.6$

We also need to specify another value, P_{13}; which is the probability of transition from state 1 to state 3. We will assume the probability of transition from state 1 to state 3 is 1; $P_{13} = 1$. All rates are in units of distance as in Model One.

1. <u>There is No Defensive Anti-Tank Weapon</u>

Table V gives the probability that at least one tank gets through the mine field for various mine field depths that are obtained from Equation (4.4),

TABLE V

Probability that at Least One Offensive Tank Gets Through the Mine Field Successfully

D	$P\{N(D) \geq 1\}$
300	0.4965
400	0.3430
500	0.2310

D, depth of mine field in meters.

Equations (4.5) and (4.6) indicate that if a defender wants to reinforce his defensive strength with mine fields

(make $P\{N(D) \geq 1\}$ smaller), there is no effective difference between increasing the rate of mines or lengthening the depth of the mine field.

2. <u>There is One Defensive Anti-Tank Weapon</u>

TABLE VI

Probability that at Least One Tank Gets Through The Mine Field Successfully Depends on Speed and Depth of Mine Field

D	Speed	$P\{N(D) \geq 1\}$
300	8	0.3914
	12	0.3885
	16	0.3859
500	8	0.1800
	12	0.1788
	16	0.1776

D, depth of the mine field in meters.

Speed: Speed of the tank in kilometers per hour.

The results of Table VI are obtained from the Equation (4.8). Note that as the speed of the offensive tanks increase, the $P\{N(D) \geq 1\}$ decreases. Increasing the speed of the offensive tanks causes the firing rate in duel to decrease from $\lambda_A = \lambda_T = 0.037$ rounds per minute to $\lambda_A = \lambda_T = 0.01875$ rounds per minute.

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Let A be the sum of all the probabilities of the events in (4.7) in which Y(D) = 0.

Let B be the sum of all the probabilities of the events in (4.7) in which Y(D) = 3. Table VII gives values for A and B for various different $\lambda_A = \lambda_T$ with all other parameters as in (5.1).

TABLE VII

Change of Probability Depends on Speed

λ_A	λ_T	A	B
0.037	0.037	4.97×10^{-9}	0.39128
0.0333	0.0333	1.89×10^{-6}	0.39063
0.0299	0.0299	6.30×10^{-6}	0.38991

λ_A, λ_T; fire rate in rounds per minutes

Note that A increases as the fire rate decreases and B decreases as the fire rate decreases, as is expected. Hence, it appears that the reason P $\{N(D) \geq 1\}$ decreases as the offensive tank speed increases is that for the range of parameters we are interested in A is very small relative to B.

Table VIII investigates the sensitivity of the five basic parameters in the model with a mine field depth of 300 meters and initial values of parameter as in (5.1).

TABLE VIII

Change of Percent of Probability

%	P_T	λ_T	R	P_A	λ_A
10	2.69	2.69	-10.50	- 2.68	- 2.68
20	5.01	5.01	-20.13	- 5.24	- 5.24
30	7.05	7.05	-28.91	- 7.67	- 7.67
40	8.83	8.83	-36.87	-10.00	-10.00
50	10.41	10.41	-44.04	-12.22	-12.22

%; increasing percent of value of parameter

P_T; hit probability of an offensive tank,

λ_T; fire rate of an offensive tank,

R; rate of mines in the mine field

P_A; hit probability of a defensive anti-tank weapon.

The value in a cell of TABLE VIII shows the change percentage of initial probability that at least one offensive tank gets through the mine field successfully due to the corresponding change in the parameter value. Positive values indicate the probability $P\{N(D)\geq 1\}$ is increased and negative values mean decreased $P\{N(D) \geq 1\}$.

Table VIII suggests that if a duel begins between two offensive tanks and one defensive anti-tank weapon,

then the rate of mines in the mine field is the most important parameter to effect the result. It indicates that a defender who wants to decrease the probability of an offensive tank crossing the mine field should make his mine field with as high a value of rate of mines as possible.

VI. CONCLUSIONS AND RECOMMENDATIONS

As one of the best means of defense against an enemy's quick attack, the mine field which is possibly defended by an anti-tank weapon, is studied. Some formulae of simple models have been derived and numerical evaluations and analysis are provided for some cases for further interpretation.

The model of this thesis used many assumptions. Some assumptions are, the mines in the preinstalled mine field act their characteristics with 100 percent reliability; the duel between offensive tanks and defensive anti-tank weapon is ended at the time when an offensive tank gets through the mine field successfully; there are unlimited ammunition supplies; and the damage of offensive tank due to anti-tank weapon fire is classified only killed or not killed.

Also, this thesis has considered in detail only two models; however, many other models can be formulated as continuous time Markov chain models. In these cases, the probability that at least one tank successfully travels through the mine field can be thought as the probability that the continuous time Markov chain is in the particular set of states at a finite time D (as in 4.7). The analytic expression for this probability will in general not be

simple; however, one can always write down a system of differential equations for it which can be solved numerically. If someone wants to make a model for a special area defense, he can use these models possibly with modifications to meet his needs. It may be more useful to combine these models with an air defense model for some special area.

In conclusion, these models are simple and some assumptions are not real. However, they can provide insight to evaluate the conventional preinstalled mine field and to evaluate the strength of defense with mine field against enemy's quick attack.

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